Mechanical Analogy

In the basic equations of PSO the φ coefficient is usually interpreted as a social/cognitive one, or a confidence coefficient. And it is then quite « normal » it can be modified at each time step. Now, if we are looking for a deterministic version, it is interesting to find an interpretation in which φ can be easily understood as a constant. The model is here a classical oscillating unit mass (the particle) attached to a spring.

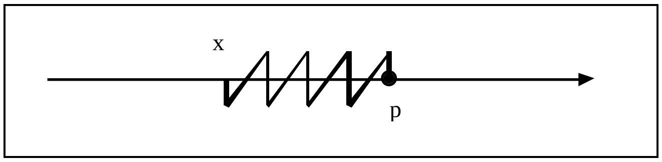

Figure 1. The virtual spring between a particle and its objective.

The coordinate of the mass is *x*, the fixation point is on *p*. Then, at each time *t*, the acceleration at time $t+\Delta t$ is $w(t+\Delta t) = -\varphi\left(x(t)-p\right)$. Let $v(t)$ be the velocity of the particle at time *t*. We have immediately $\frac{v(t+\Delta t)-v(t)}{\Delta t} = w(t+\Delta t)$ and $\frac{x(t+\Delta t)-x(t)}{\Delta t} = v(t+\Delta t)$. So, finally, for discrete time steps ($\Delta t = 1$), we have the following system

$$\begin{cases} w(t+1) = -\varphi\left(x(t)-p\right) \\ v(t+1) = v(t) + w(t+1) \\ x(t+1) = x(t) + v(t+1) \end{cases} \qquad \textbf{Equ. 1}$$

and we retrieve easily the basic simplified PSO equations

$$\begin{cases} v(t+1) = v(t) + \varphi\left(p - x(t)\right) \\ x(t+1) = x(t) + v(t+1) \end{cases} \qquad \textbf{Equ. 2}$$

The only difference is that φ is now seen as the ridigity coefficient of a virtual spring between the particule and its objective. Another point of view could be to see the particle as more or less « intelligent » and applying the following rules :
« The more I am far ahead the objective, the more I have to speed up, the more I am beyond it, the more I have to slow down».

But let us continue with our mechanical analogy. We now add the usual damping part depending on the velocity (the particle is getting « tired »), and we define the pulsation $\omega = \sqrt{\varphi}$. We obtain then :

$$w(t+\Delta t) = -2\lambda\omega v(t) + \omega^2\left(x(t)-p\right) \qquad \textbf{Equ. 3}$$

the Equ. 2 become

$$\begin{cases} v(t+1) = (1-2\lambda\omega)v(t) + \omega^2\left(p - x(t)\right) \\ x(t+1) = x(t) + v(t+1) \end{cases} \qquad \textbf{Equ. 4}$$

and we find the well known solution

$$x(t) = p + e^{-\lambda\omega t}\left(c_1 e^{\omega t\sqrt{\lambda^2-1}} + c_2 e^{-\omega t\sqrt{\lambda^2-1}}\right) \qquad \textbf{Equ. 5}$$

where c_1 and c_2 are depending on the initial conditions. It is yet another particular case of the general five parameters model, near of we have called Constriction Type 1''. We have indeed here :

$$\begin{bmatrix} v(t+1) \\ p - x(t+1) \end{bmatrix} = \begin{bmatrix} \alpha & \beta\varphi \\ -\gamma & \delta - \eta\varphi \end{bmatrix}\begin{bmatrix} v(t) \\ p - x(t) \end{bmatrix} \qquad \textbf{Equ. 6}$$

with

$$\begin{cases} \alpha = 1 - 2\lambda\omega \\ \beta = 1 \\ \gamma = 1 - 2\lambda\omega \\ \delta = 1 \\ \eta = 1 \end{cases}$$

Equ. 7