

PSO: A few words about constriction

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Let us consider the "physically plausible" constricted PSO type, that is to say the case where the new position is found just by adding the velocity "as is":

For a given particle and a given dimension d , we have

$$\begin{cases} v_d(t+1) = \alpha \left(v_d(t) + \text{rand}(0, \varphi_1) (p_{i,d}(t) - x_d(t)) + \text{rand}(0, \varphi_2) (p_{g,d}(t) - x_d(t)) \right) \\ x_d(t+1) = x_d(t) + v_d(t+1) \end{cases}$$

In this case, for $\varphi = \varphi_1 + \varphi_2 > 4$, we have the following constriction coefficient

$$\chi = \frac{2\kappa}{\left(\varphi - 2 + \sqrt{\varphi^2 - 4\varphi} \right)}$$

and the convergence criterion (eigenvalues norms smaller than 1) says us that we must have

$$\begin{cases} \kappa \in]0, 1[\\ \alpha = \frac{1 + \chi(\varphi - 2)}{\varphi - 1} \end{cases}$$

However, a combination of partial proofs and heuristic reasoning suggest that we should also have, more precisely

$$\alpha = e - 2 \cong 0.718$$

This immediately implies an interesting relation between κ and φ :

$$\kappa = ((e - 2)(\varphi - 1) - 1) \left(1 + \frac{\sqrt{\varphi^2 - 4\varphi}}{\varphi - 2} \right)$$

which gives us a "rule" to choose κ and φ . For example:

κ	φ
0.577	4
0.8	4.1
0.919	4.2
1	4.276525

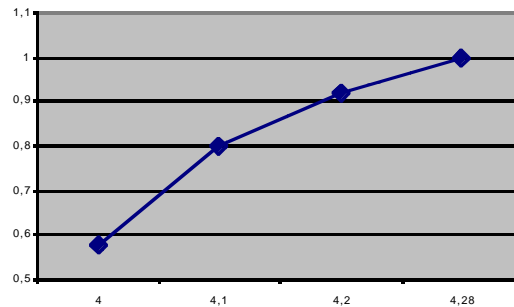


Figure 1. κ vs φ under the conjecture $\alpha = e - 2$.

In particular, it gives us a lower bound for κ (0.577) and a upper bound for φ (4.276). For the moment this can be seen as a rule of thumb to choose a pair (κ, φ) , but, interestingly, it is quite consistent with what most of authors do use. Of course, it does not mean you can't have convergence with some other values (say (0.8, 4.2)), but it suggests that some specific pairs may be better than some others.