Examples

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March 9, 2015

Reference: Scilab codes availables at http://clerc.maurice.free.fr/Maths/index.htm, Combinatorial tools section.

Let us suppose we have two permutations of the integers $(1, \ldots, 6)$:

$$p_1 = (5, 3, 1, 2, 6, 4) \tag{1}$$

$$p_2 = (3, 2, 1, 4, 6, 5) \tag{2}$$

Permutation2 - Permutation1

We want to find a minimal list of transpositions $\tau_{1,2}$, so that, when applying it to p_1, the result is p_2. In short, $p_2 = p_1 + \tau$, i.e. $\tau = p_2 - p_1$

Here are the Scilab steps by using the codes given in the reference:

```
-->p1=[5,3,1,2,6,4];
-->p2=[3,2,1,4,6,5];
-->tau12=permutDecompCayley(p1,p2,[])
tau12 =
1. 1. 1.
6. 4. 2.
```

which means

- switch values of ranks 1 and 6 ($(5, 3, 1, 2, 6, 4) \Longrightarrow (4, 3, 1, 2, 6, 5)$)
- switch values of ranks 1 and 4 ($(4, 3, 1, 2, 6, 5) \Longrightarrow (2, 3, 1, 4, 6, 5)$)
- switch values of ranks 1 and 2 ($(2, 3, 1, 4, 6, 5) \Longrightarrow (3, 2, 1, 4, 6, 5)$)

Let $p_0 = (1, 2, 3, 4, 5, 6)$ bte he "identity" permutation.

The permutation p_1 can be seen as the result of a sequence of transpositions $\tau_{0,1}$ applied to p_0.

Whe have:

```
-->p0=[1,2,3,4,5,6];
-->tau01=permutDecompCayley(p0,p1,[])h
tau01  =
 1.    1.    1.    1.    1.
 3.    2.    4.    6.    5.
```

i.e.

$$\tau_{0,1} = \left((1,3)\,,(1,2)\,,(1,4)\,,(1,6)\,,(1,5)\right) \tag{3}$$

From this point of view, the formula 1 is a compact representation of the formula 3. And the compact representation of $\tau_{1,2}$ is given by a permutation. But which one? We have:

```
-->p2minusp1=transpoApply(p0,tau12)
p2minusp1 =
2. 4. 3. 6. 5. 1.
i.e.
```

$$p_2 - p1 = (2,4,3,6,5,1) \tag{4}$$

Permutation1 + Permutation2

Similarly, p_2 can also be seen as a compact representation of a list of transpositions, say $\tau_{0,2}$. Therefore, applying successively $\tau_{0,1}$ and then $\tau_{0,2}$ is equivalent to a sequence of transpositions whose compact representation is a permutation. We have:

```
-->tau02=permutDecompCayley(p0,p2,[])
tau02 =
1. 5.
3. 6.
-->p12=transpoApply(p1,tau02)
p1plusp2 =
1. 3. 5. 2. 4. 6.
```

i.e.

$$p_1 + p_2 = (1,3,5,2,4,6) \tag{5}$$

Note that this "addition" is usually not commutative. We indeed have:

```
-->p2plusp1=transpoApply(p2,tau01)
p2plusp1 =
6. 1. 3. 2. 5. 4.
```

i.e.

$$p_2 + p_1 = (6,1,3,2,5,4) \tag{6}$$